Government of Puducherry
Directorate of School Education
Competitive Examination for Empanelment of Guest Lecturers-2019

QUESTION BOOKLET

SUBJECT: MATHEMATICS

ROLL NO: ....

Time: 2:00 Hours   **Total Marks: 90**

INSTRUCTIONS TO CANDIDATES

1) Write Roll Number in the space provided in this Question Booklet above.

2) The Candidate should check the question paper that question paper consists of all the pages and that it is not torn.

3) Please write the Roll No. etc., on the OMR sheet using Black Ball point pen only.

4) The candidate shall use only **"BLACK BALL POINT PEN"** for marking the answers.

5) There are **90 questions**. For every correct answer 1 (One) mark will be awarded and for each wrong answer 0.25 mark will be deducted.

6) Use of eraser/ whitener/ correction fluid is prohibited on OMR answer sheets.

7) Use of calculators, cell phones, logarithmic table, electronic gadgets etc., is strictly prohibited.

8) For any rough work, use the rough page given at the end of the Question Booklet.

9) The candidate has to deposit the original copy of OMR sheet along with the Hall Ticket after the completion of examination and may carry the duplicate OMR (impression) sheet for reference.

10) No candidate is allowed to leave the examination centre premises till the completion of the entire examination process.

11) Please avoid marking of answers on Question Booklet.

12) The questions in English version alone will be taken as authentic though questions are given in other languages for the convenience of the candidates.

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MATHEMATICS

1. If $G=\{1, -1, i, -i\}$ is a group w.r.t complex multiplication then order -1 and i are
   a) 2,4   b) 1,1   c) 4,4   d) 2,3

2. If $S = \{1,2,3,4\}$ then the number of permutations on S is
   a) 6   b) 8   c) 4   d) 24

3. The set of all real numbers under the usual multiplication is not a group since
   a) Multiplication is not a binary operation
   b) Multiplication is not associative
   c) Identity element does not exist
   d) Zero has no inverse

4. If $(G,.)$ is a group then $(ab)^{-1}$ is
   a) $a^{-1}b^{-1}$ b) ab c) $b^{-1}a^{-1}$ d) ba

5. If $(z, *)$ is a group with $a*b = a+b+1$ $\forall a , b \in Z$ then the inverse of a is
   a) 0   b) -2   c) a-2   d) -a-2

6. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda} x +4 = 0$ is such that $\sin\theta = 1/3$, then the value of λ is
   (a) 5/3   (b) -3/5   (c) 0   (d) - 4/3

7. The point which is farthest on the sphere $x^2 + y^2 + z^2 = 144$ from the point (2,4,4) is
   (a) (3,6,6)   (b) (0,0,0)   (c) (1,1,1)   (d) (-4,-8,-8)

8. If the line $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies exactly on the plane $2x - 4y +z = 7$, then the value of k is
   (a) 7   (b) -1   (c) 0   (d) none

9. The angle between the planes $2x - y +3z = 6$ and $x + y +2z = 7$ is
   (a) 0°   (b) 30°   (c) 45°   (d) 60°

10. The distance of a point (1,-2,3) from the plane $x - y + z = 5$ and parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is
    (a) 1   (b) 7   (c) 3   (d) 13

11. If $\text{Arg} \frac{z-1}{z+1} = \frac{\pi}{4}$, then the locus of z is
    a) Circle   b)Straight line   c)Ellipse   d)None

12. The imaginary part of $w = \log z$ is
    a) x+iy   b) $\frac{1}{2} \log(x^2 + y^2)$ c) $\tan^{-1}(y/x)$ d) None

13. If $f(z) = e^z$, then
    a) f(z) is entire function
    b) Analytic only at $z = 0$
    c) f(z) is nowhere analytic
    d) None

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14. The Residue of $f(z) = \frac{z}{(z-1)(z-2)}$ at $z = 1$ is

   a) -1   b) 0   c) 1   d) 2

15. The mapping $w = z + \alpha$ is

   a) Magnification   b) Rotation   c) Translation d) None

16. Which of the following functions is not differentiable

     a) $|x|$ b) $(x+3)^4$ c) $\sin(x)$ d) $mx+b$

17. If $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ cut each other orthogonally if

    a) $-1/a - 1/b = 1/c - 1/d$
    b) $1/a + 1/b = 1/c - 1/d$
    c) $1/a + 1/b = 1/c + 1/d$
    d) None

18. The curvature of the cardioids $r = a(1+\cos\theta)$ at $\theta = 0$ is

    a) $3/4a$ b) $3/a$ c) a d) a^2

19. The equation of tangent to the curve $y = x^3 - x + 3$ at $(1,3)$ is

    a) $x - 2y + 1 = 0$ b) $2x - y + 1 = 0$

    c) $x = 0$ d) $y = 0$

20. n^{th} derivative of $y = \log(2x+3)$ is

    a) $y^{(n)} = \frac{n!2^n}{(2x+3)^n}$

    b) $y^{(n)} = (-1)^{n-1} \frac{(n-1)! \, 2^n}{(2x+3)^n}$

    c) $y^{(n)} = \frac{(n-1)! \, 3^n}{(2x+3)^n}$

    d) none

21. Evaluate the integral $\int_0^\infty e^{-x^2} dx$

   (a) $\frac{\sqrt{\pi}}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) none

22. The value of $\int_0^2 \int_0^x (x + y) dx \, dy$ is

   (a) 3/4   (b) 3/8   (c) 3/5   (d) 0

23. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 \theta \, d\theta$ is

   (a) $5\pi/32$ (b) 5/32   (c) $\pi/32$ (d) none

24. The value of the integral $\int x^2 \log_e x \, dx$ is

   (a) $\frac{x^3}{3} \log_e(x) - \frac{x^3}{9} + c$

   (b) $\frac{x}{3} \log_e(x) - \frac{x}{9} + c$

   (c) $\frac{x^2}{3} \log_e(x) + x^2 + c$

   (d) none

25. The value of $\Gamma(1/2)$ is

    (a) 0   (b) $\sqrt{\pi}$ (c) $\frac{\pi}{2}$ (d) π

26. If $A=\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$ then the smallest positive integer n such that $A^n = I$

    a) 1   b) 2   c)4   d) 6

27. The Dimension of the vector space of complex numbers over real numbers is

    a) 2   b) 1   c)0   d)3

28. The Sum and product of Eigen values of $A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

    a)2,-2   b) 0,0   c) 3,1   d)-1,0

29. The rank of the matrix $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is

    a) 2   b) 1   c) 3   d) 0

30. If the two columns of a matrix are identical then its determinant is

    a)2   b) 0   c) 1   d) -1

31. Which of the following propositions is a tautology
    a) $(P \vee Q) \to Q$
    b) $P \vee (Q \to P)$
    c) $P \vee (P \to Q)$
    d) Both (b) and (c)

32. $P \to (Q \to R)$ is Equivalent to
    a) $(P \wedge Q) \to R$
    b) $(P \vee Q) \to R$
    c) $(P \vee Q) \to 7R$
    d) None

33. Hasse diagrams are first made by
    a) A.R. Hasse
    b) Hemat Hasse
    c) Dennis Hasse
    d) T.P.Hasse

34. Which of the following relation is partial ordering as well as equivalence relation
    a) =   b) <   c) >   d) not equal

35. The relation $\leq$ is partial order if it is
    a) Reflexive, symmetric, transitive
    b) Reflexive, anti -symmetric, transitive
    c) Reflexive only
    d) None

36. The Mean and Variance of Binomial distribution
    a) np, npq
    b) $\sqrt{np}$, npq
    c) n/p, n/q
    d) *none*

37. The variance of uniform distribution defined over [a,b]
   a) $\frac{1}{12}(b-a)^2$
   b) $(b-a)^2$
   c) $\frac{a+b}{2}$
   d) none

38. The probability of a leap year selected at random contains 53 Sundays
   a) 1/7 b) 2/7 c) 3/7   d) 1

39. If X and Y are Independent random variables then E(XY) is
   a) E(X). E(Y)
   b) E(X)+E(Y)
   c) E(X)/E(Y)
   d) None

40. Inflexion points of normal distribution are
   a) $\mu \pm \sigma$ b) μ and σ^2 c) 0 and 1   d) none

41) The augmented matrix in gauss Jordan method is reduced to
   a) Row Echelon form
   b) Column Echelon form
   c) Matrix Echelon form
   d) Augmented form

42) The convergence of which of the following method is sensitive to starting value
   a) False Position
   b) Gauss Seidal Method
   c) Newton Raphson Method
   d) All of these

43) If we solve $x^2-2=0$ by Newton Raphson method and if initial guess is $x_0=1.0$ then the next subsequent estimate of x(i.e x_1) is

   a) 1.414   b) 1.5   c) 2.0   d) none

44) If f(x) is constant then $\Delta f(x)$ is

   a) 0   b) x   c) 1   d)none

45) The order of error in the Simpson's rule for numerical integration with step size h is

   a) h   b) h^2 c) h^3 d) h^4

46. In a M/M/1 Queueing model if λ=10 customers/m and service time is 5 minutes then the number of customers in the system is
   a) 0   b) 5   c) 4.5   d) 4

47. The stability condition of M/M/c Queues model is

   a) $\frac{\lambda}{c\mu}$ b) $\frac{\lambda}{\mu}$ c) $\frac{\lambda^2}{c\mu}$ (d) none

48. Relation between L_s and L_q in Little's formula is

   a) $L_s = L_q + \frac{\lambda}{\mu}$ b) $L_s = L_q - \frac{\lambda}{\mu}$ c) $L_s = L_q + \frac{\mu}{\lambda}$ d)none

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49. The Expected time of an activity in PERT is

a) $t_e = \dfrac{t_0 + 4t_m + t_p}{6}$ b) $t_e = \dfrac{t_0 - 4t_m + t_p}{6}$ c) $t_e = \dfrac{t_0 + 4t_m + t}{6}$ d) none

50. The number of basic feasible solutions of
$2x_1 + 6x_2 + 2x_3 + x_4 = 3;\ 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2,\ x_i \geq 0$
a) 5 b) 6 c) 2 d) 4

51. The particular solution of $(D^2+1) = \sin x$ is
a) $\sin x + \cos x$ b) $\dfrac{-x}{2}\cos x$ c) $\dfrac{x}{2}\sin x$ d) $\sin x$

52. The general solution of $\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} = 0$ is
a) $A + Be^{-x}$ b) $Ae^x + B$ c) $Ae^x + Be^{-x}$ d) $A + Bx$

53. Two independent solutions of $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + by = 0$ are
a) e^{-2x}, e^{-3x} b) e^{2x}, e^{3x} c) e^{-2x}, e^{3x} d) e^{2x}, e^{4x}

54. The Laplace transform of $L[t^3]$ is

a) $\dfrac{6}{s^4}$ b) $\dfrac{12}{s^3}$ c) $\dfrac{1}{s^3}$ d) $\dfrac{4}{s^3}$

55. The Laplace transform of $L[sinhat]$ is

a) $\dfrac{a}{s^2 - a^2}$ b) $\dfrac{a}{s^2 + a^2}$ c) $\dfrac{1}{s^2 + a^2}$ d) $\dfrac{s}{s^2 + a^2}$

56. If $xzp + yzq = xy$ then the general solution is
a) $\varphi\left(\dfrac{x}{y}, \dfrac{y}{z}\right) = 0$
b) $\varphi(x + y + z, xyz) = 0$
c) $\varphi\left(xz, \dfrac{y}{z}\right) = 0$
d) None

57. The complete integral of $z = pq$ is
a) $z = (x+a)(y+b)$ b) $z = x+y+a$
c) $z = x + a/y + b$ d) None

58. The complementary function of $(D^2 - 2DD' - 15D'^2)Z = 12xy$ is
a) $C.F = \varphi_1(y+5x) + \varphi_2(y-3x)$
b) $C.F = \varphi_1(y+x) + \varphi_2(y-x)$
c) $C.F = \varphi_1(y-2x) + \varphi_2(y+2x)$
d) None

59. The value of the wronskian of the function $x^2, 3x+2, 2x+3$ is
a) 0 b) $2+x$ c) $10+x^2$ d) -10

60. The Laplace equation in 3 dimension is
a) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$
b) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$
c) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$
d) $\dfrac{\partial^2 u}{\partial x\, \partial yz} = 0$

61. $Lim_{n\to\infty}\left(1-\dfrac{1}{n^2}\right)^n$ Equals

   a) 1 b) $e^{-\frac{1}{2}}$ c) e^{-2} d) e^{-1}

62. Let $f: R \to R$ be continuous function and $f(x+1) = f(x)$ for all $x \in R$. Then
   a) f is bounded above, but not bounded below
   b) f is bounded above and below, but may not attain its bounds
   c) f is bounded above and below and f attains its bound
   d) F is uniformly continuous

63. Let $f: R \to R$ R be a twice continuously differentiable function with
   $f(0) = 0, f(1) = 0$ and $f'(0) = 0$ Then
   a) f'' is the zero function
   b) f''(0) is Zero
   c) f''(x)=0 for some x∈(0,1)
   d) f'' vanishes

64. Let $S_n = \displaystyle\sum_{k=1}^{n}\dfrac{1}{k}$ which of the following is true
   a) $S_{2^n} \geq \dfrac{n}{2}$ for every n≥1
   b) $\{S_n\}$ is bounded
   c) $|S_{2^n} - S_{2^{n-1}}| \longrightarrow 0$ as n $\longrightarrow \infty$
   d) $\dfrac{S_n}{n} \longrightarrow 1$ as n$\longrightarrow \infty$

65. The limit, $\lim_{n\to\infty}\dfrac{1}{x}\displaystyle\int_{x}^{2x}e^{-t^2}dt$
   a) Does not exist
   b) Is finite
   c) Exist and equals 1
   d) Exist and equals

66. If $\vec{r}=x\vec{i} + y\vec{j} + z\vec{k}$, then ∇r^n is
   a) $nr^{n-2}\vec{r}$ b) 0 c) $(n-1)r^{n-1}\vec{r}$ d)none

67. The directional derivative of $\varphi = x^2yz + 4xz^2 + xyz$ at (1,2,3) in the direction of
   $2\vec{i} + \vec{j} - \vec{k}$ is
   a) $\vec{i}+\vec{j}+\vec{k}$ b)$86/\sqrt{6}$ c) 1 d) 0

68. $div(\vec{r})$ is
   a) 3 b) 0 c) -1 d) 1

69. If $\overrightarrow{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then $\int_c \overrightarrow{F}.\overrightarrow{dr}$ from (0,0,0) to (1,1,1)
   along the curve x = t, y = t², z = t³ is
   a) 2 b) 1 c) 5 d) 0

70. A vector field with a vanishing curl is called as
   a) Irrotational b) Solenoidal c) Rotational d) Cycloidal

71. Find the Sum and Product of roots of $x^3 - 3x^2 + 4x - 10 = 0$
    (a) 3,10   (b) 1,10   (c)4,10   (d)-3,-10

72. If the roots of the equation $2x^2 - 5x + b = 0$ are in the ratio of 2:3, then find the value of b?
    (a) -1   (b) 3   (c) 4   (d)-1

73. The sum of the squares of two consecutive positive integers exceeds their product by 91. Find the integers?
    (a) 5,10   (b) 2,8   (c) 3,9   (d)9,10

74. If x =-1 is zero with multiplicity 2 of the equation $x^4 + x^3 + x^2 + kx + k - 1 = 0$ then the value of k is
    (a) 3   (b) -1   (c)2   (d) 0

75. The number of Positive roots of $x^4 + 3x^3 + 2x^2 + x + 1 = 0$
    (a) 0   (b) 1   (c) 4   (d) 3

76. If $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A} \times \vec{B}$ is
    a) Solenoidal   b) Irrotational   c) Scalar   d) none

77. The unit normal to the surface $\varphi(x, y, z) = c$ at the point (x, y, z) is
    a) $\frac{\nabla\varphi}{|\nabla\varphi|}$ b) $\nabla\varphi$ c) $\nabla^2\varphi$ d) 0

78. The partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{5\partial^2 z}{\partial y^2} = 0$ is classified as
    a) Elliptic
    b) Parabolic
    c) Hyperbolic
    d) None

79. The PDE by eliminating the arbitrary constants a and b from z=ax+by is
    a) Z=p+q   b)Z=px-qy
    c) Z=p²x+q²y   d) Z=px+qy

80. The series $\sum \frac{1}{n^p}$ is convergent is
    a) p≥1   b) p≤1   c) p> 1   d) p<1

81. What does the moment of the force measure?
    a) The tendency of rotation of the body along any axis
    b) The moment of inertia of the body about any axis
    c) The couple moment produced by the single force acting on the body
    d) The total work is done on the body by the force

82. The tendency of rotation of the body along any axis is also called _____
    a) Moment of inertia
    b) Moment of couple
    c) Torque
    d) Force

83. The moment is the cross product of which two vectors?
   a) Force and Radius vectors
   b) Radius and Force vectors
   c) Force and Radius scalars
   d) Radius and Force scalars

84. The centre of gravity is the ratio of _____ to _____
   a) The product of centroid and weight to the total weight
   b) The addition of centroid and weight to the total weight
   c) The subtraction of centroid and weight to the total weight
   d) The product of centroid and weight to the total mass

85. The measure of the body's resistance to angular acceleration is called _____
   a) Mass moment of inertia of a body
   b) Volume moment of inertia of a body
   c) Line moment of inertia of a body
   d) Gauss moment of inertia of a body

86. β_2 of normal distribution is
   a) 0　　　　b) 3　　　　c) 1　　　　d) 2

87. A coin is tossed until a head appears. What is the Expectation of the number of tosses required
   a) 1/2　　　b) 2　　　c) 3　　　d) 0

88. If X and Y are independent variables then Cov(X,Y) is
   a) 0　　　　b) -1　　　c) 1　　　d) none

89. If X is N(0,1), then E($|X|$) is

   a) $\sqrt{\pi}$　　　b) $\sqrt{\dfrac{2}{\pi}}$　　　c) $\dfrac{1}{\sqrt{\pi}}$　　　d) 0

90. The cumulants of the poisson distribution are all equal to
   a) λ　　　b) $\lambda^2 + \lambda$　　　c) λ^2　　　d) none